

# **Estereología: Un aspecto del legado científico de Luis A. Santaló**

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Girona, 06 Octubre 2011.

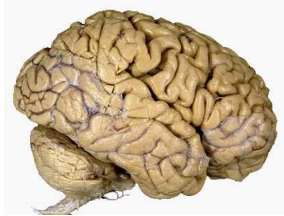
# Estereología: Ciencia y arte del muestreo geométrico

## Plan

La charla presenta una revisión informal de algunos aspectos de la Estereología.

- Motivación y breve historia.
- Un ejemplo introductorio: ¿Se pierden neuronas con la edad?
- Representación de un volumen, superficie y longitud (en tres dimensiones) en función de valores medios observables en secciones.
- Ejemplos de estimación de dichas cantidades en casos reales.

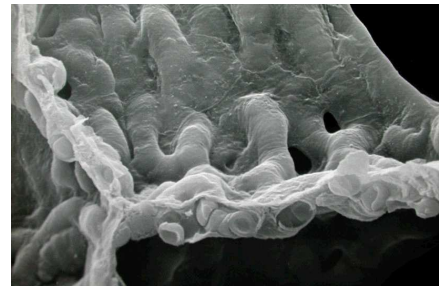
## Motivación



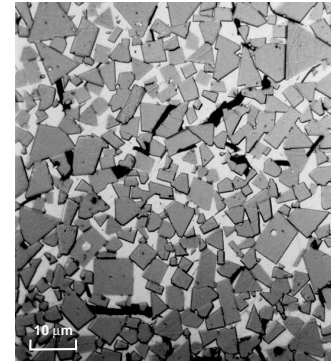
Human brain  
(USD Health)



Muscle capillary network  
(Ewald R. Weibel, Bern)



Alveolar septum of lung  
(Ewald R. Weibel, Bern)

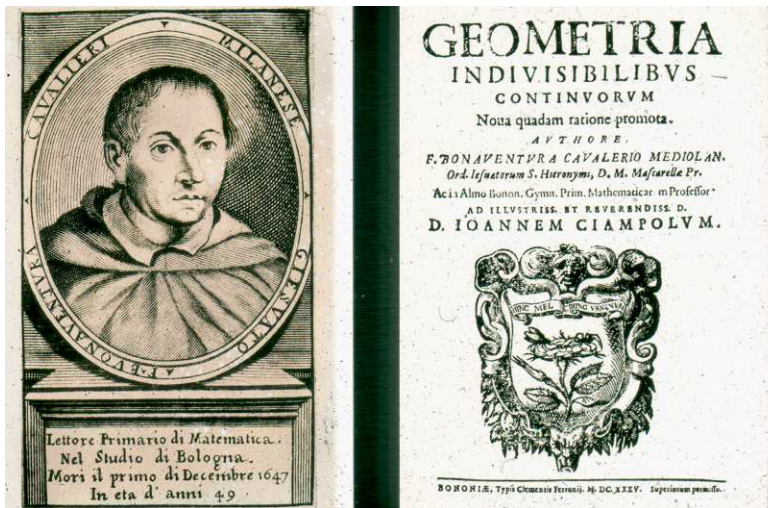


Tungsten carbide  
(L.M. Karlsson, Sweden)

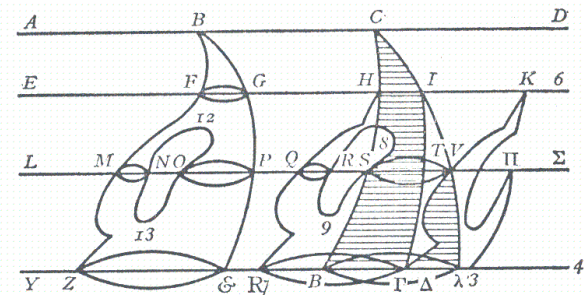
# Notas históricas

## Precedentes

- Bonaventura Cavalieri (1598–1647)



Bonaventura Cavalieri  
y su *Geometria Indivisibilibus* (1635)

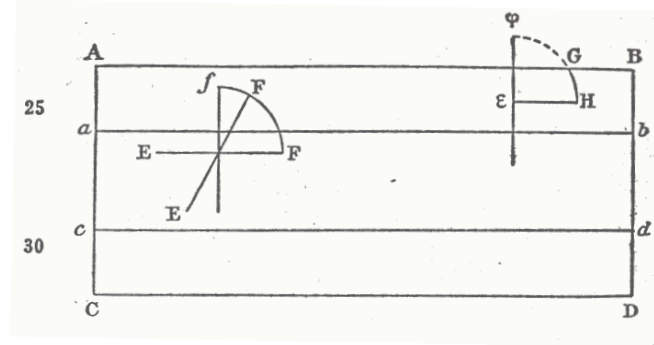


Principio de Cavalieri (1635)

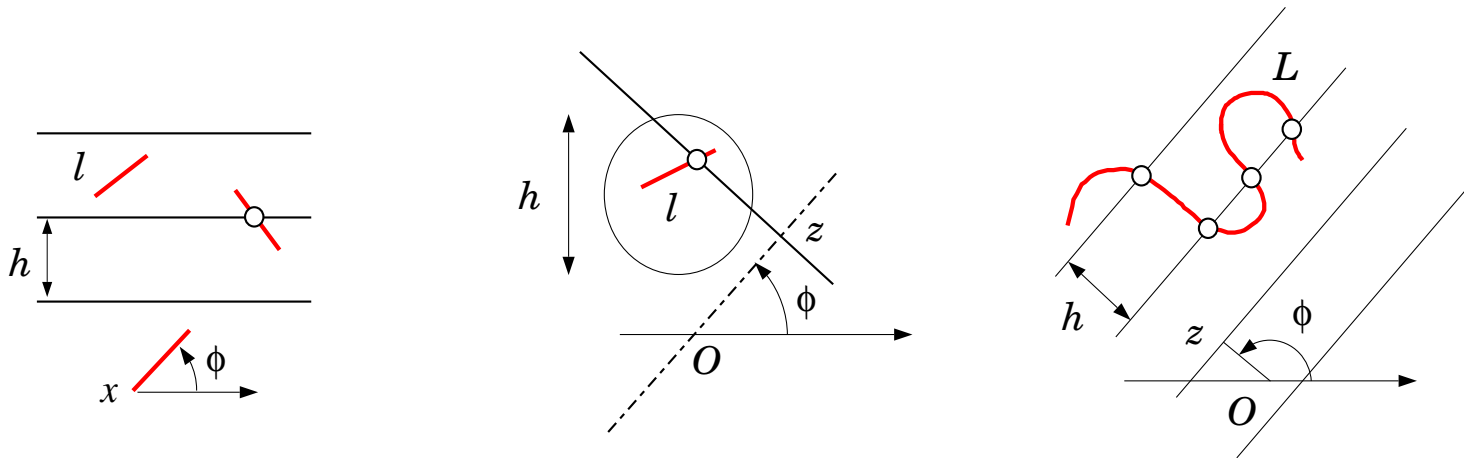
- George Louis Leclerc, Comte de Buffon (1707–1788)



Comte de Buffon (1707-1788)



Buffon's needle problem (1777)



$$\mathbb{P}(dz, d\phi) = \frac{dz}{h} \cdot \frac{d\phi}{\pi}, \quad \Rightarrow \quad \mathbb{P}(\text{line hits segment}) = \mathbb{E}I = \frac{2l}{\pi h}, \quad \Rightarrow \quad l = \frac{\pi}{2} \cdot h \cdot \mathbb{E}I,$$

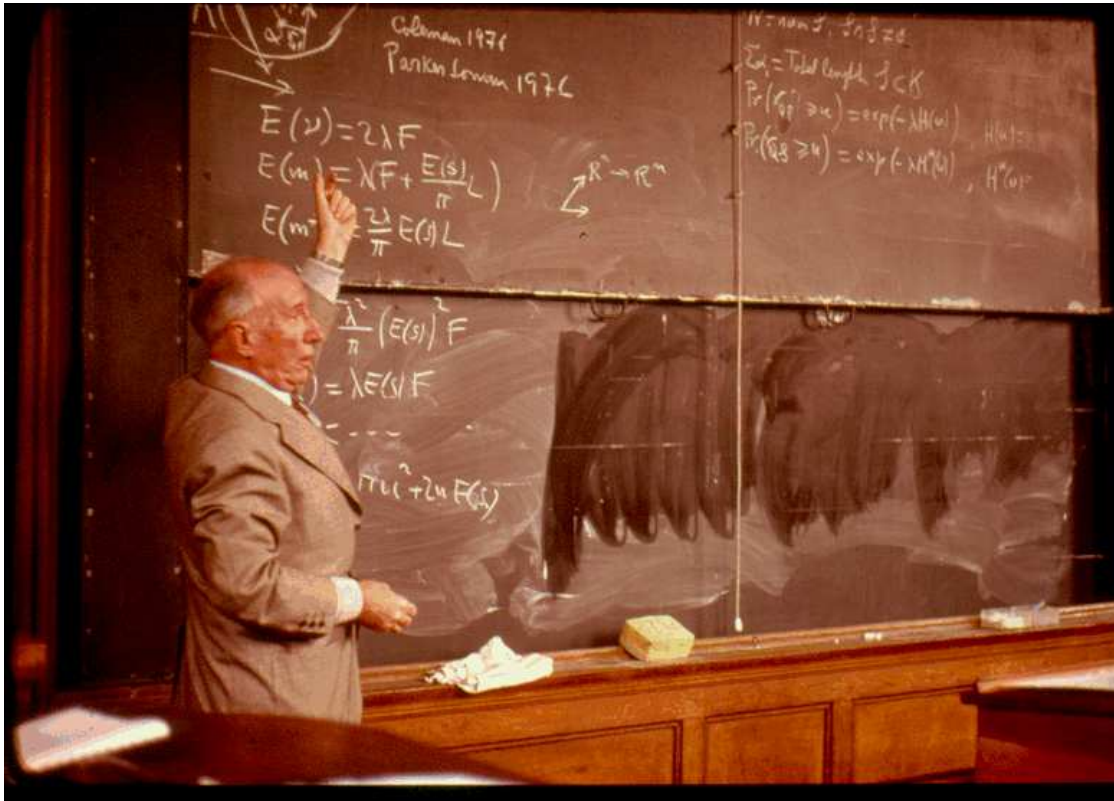
$$\text{and by the Monotone Convergence Theorem (Beppo Levi)} \quad \Rightarrow \quad L = \frac{\pi}{2} \cdot h \cdot \mathbb{E}I.$$

*Buffon Bicentenary Symposium on Geometrical Probability, Image Analysis, Mathematical Stereology, and Their Relevance to the Determination of Biological Structures. Paris, June 1977*



Miles, R.E. and Serra, J. (eds.) (1978) *Geometrical Probability and Biological Structures: Buffon's 200th Anniversary. Proceedings, Paris, June 1977*. Lecture Notes in Biomathematics No. 23. Berlin: Springer-Verlag.

Buffon Bicentenary Symposium, Paris, June 1977, (cont.)



## Desarrollo inicial de la Probabilidad Geométrica y la Geometría Integral

- Morgan W. Crofton (1826–1915)
- Elie J. Cartan (1869–1951)
- Wilhelm J. E. Blaschke (1885–1962)
  - Shiing-Shen Chern (1911–2004)
  - Hugo Hadwiger (1908–1981)
  - Bojan L. Petkantschin (1907–1987)
  - Luis Antonio Santaló Sors (1911–2001)

Books (chronological order):

- Santaló, Luis A. (1953) *Introduction to Integral Geometry*. Hermann.
- Kendall, Maurice G., and Moran, Patrick A.P. (1963) *Geometrical Probability*. Ch.Griffin.
- Santaló, Luis A. (1976) *Integral Geometry and Geometric Probability*. Addison-Wesley.
- Stoyan, Dietrich, Kendall, Wilfrid S. & Mecke, Joseph (1995) *Stochastic Geometry and its Applications*, Wiley.
- Schneider, Rolf & Weil, Wolfgang (2008) *Stochastic and Integral Geometry*. Springer.
- Naveira, Antonio M., Reventós, Agustí *et al.* (eds.) (2009) *Luis Antonio Santaló. Selected Works*. Springer.



## Desarrollo inicial de la Estereología (teoría y aplicaciones)

- Achille E.O.J. Delesse (1817–1881)
- Andrei A. Glagolev (1894–1969)
- 1963: *Fundación de la International Society for Stereology (Sociedad Internacional de Estereología)*
- 1963– : *Congresos Europeos e Internacionales de la ISS* (cada dos años, alternando)
- Roger E. Miles (1935–)
- 1977– : *Cursos y Talleres Internacionales de la ISS*

### Journals:

- *Journal of Microscopy*
- *Image Analysis and Stereology*

### Books (chronological order):

- Weibel, Ewald R. (1979,1980) *Stereological Methods*. (2 vols.). Academic Press.
- Howard, C. Vyvyan & Reed, Matt G. (2005) *Unbiased Stereology*. 2nd ed. Taylor & Francis.
- Baddeley, Adrian & Jensen, Eva B.V. (2005) *Stereology for Statisticians*. Chapman & Hall/ CRC.

4th International Conference in Stereology and Stochastic Geometry

Haus der Universität, CH-Bern, September 12-15, 1987



Back row : F. Affentranger, W. Weil, J. Hüsler, H. Carnal, M. Moss, J. Mecke, R-T. DeHoff, F. Montes, H.J.G. Gundersen, A.J. Baddeley, L.A. Santaló, E.R. Weibel, C.V. Howard, Eva B. Jensen, R. Coleman, T. Mattfeldt, B. Matérn  
Front row : R.E. Miles, Ch. Lantuejoul, R.A. Vitale, L.M. Cruz-Orive, J. Møller, R. Schneider, M. Tanemura

*4th International Conference on Stereology and Stochastic Geometry*  
CH-Bern, September 1987



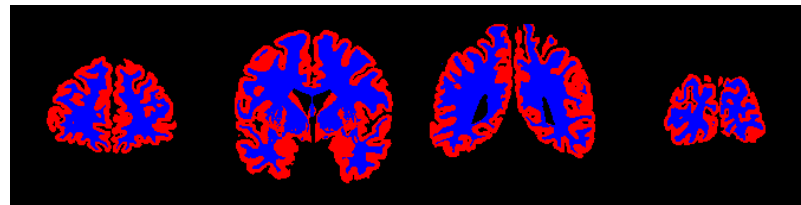
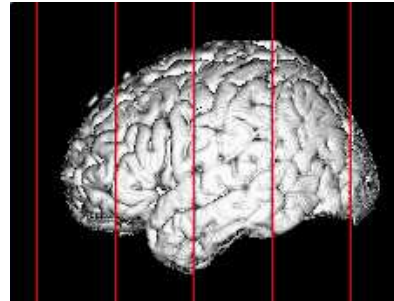
*L.A. Santaló. La Matemática: Una Filosofía y una Técnica*

Càtedra "Ferrater Mora" de Pensament Contemporani, Estudi General de Girona (U.A.B.), Novembre 1991.

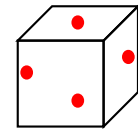
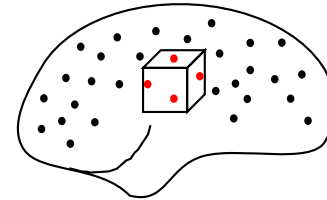
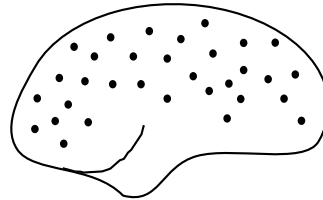
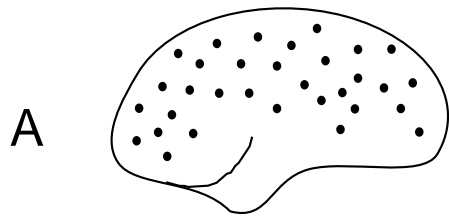
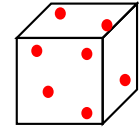
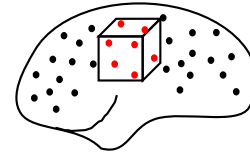
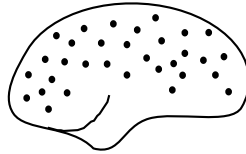
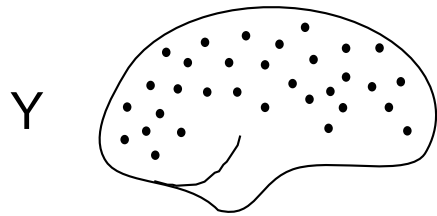


## ¿Se pierden neuronas con la edad?

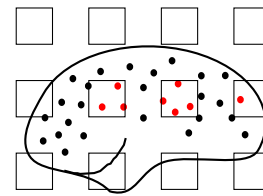
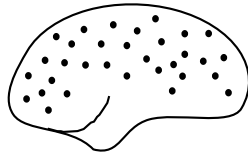
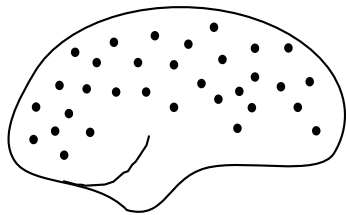
Herbert Haug (1985) *Nervenheilkunde* 4, 103–109.



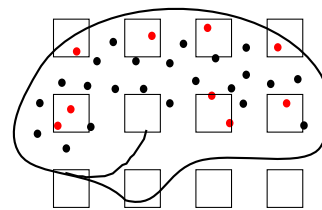
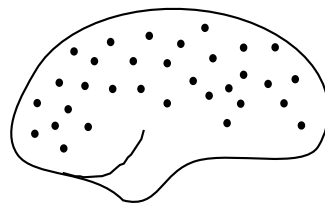
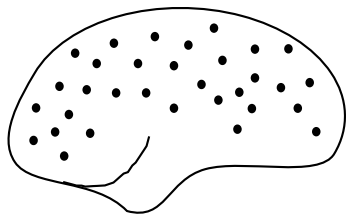
(V. McNulty *et al* (2000) *J. Computer Assisted Tomography* 24, 466–477)



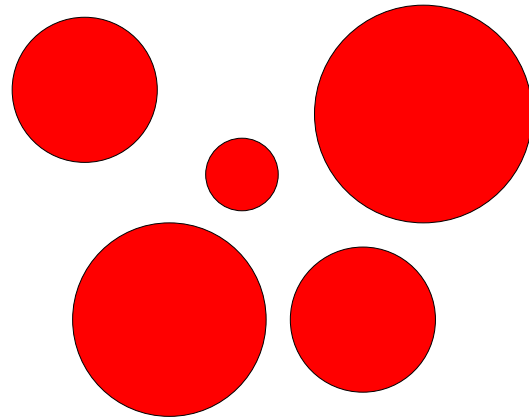
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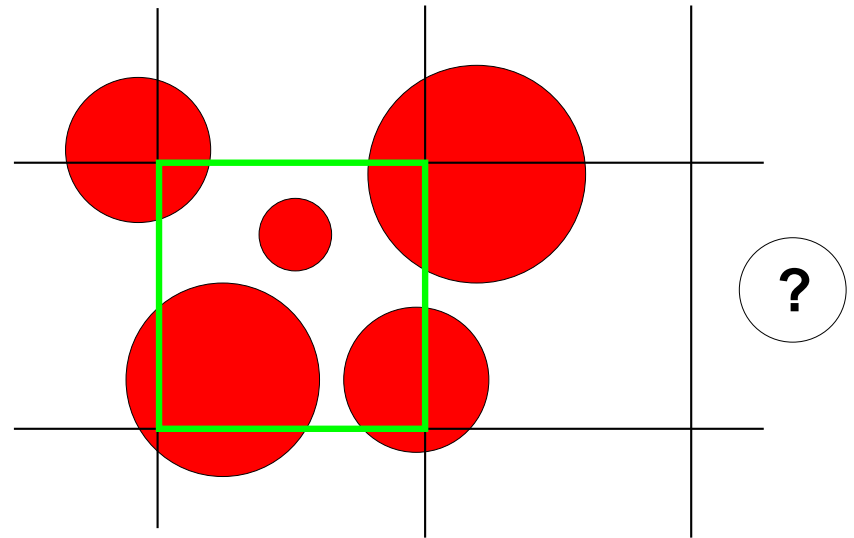
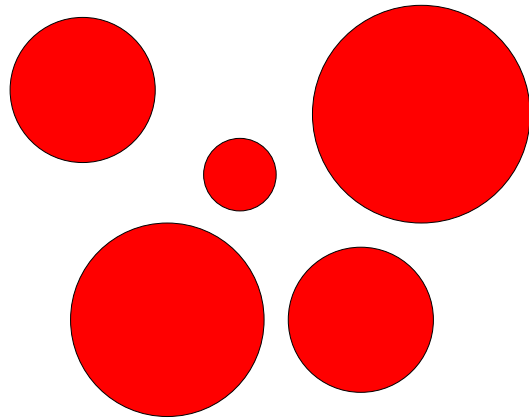
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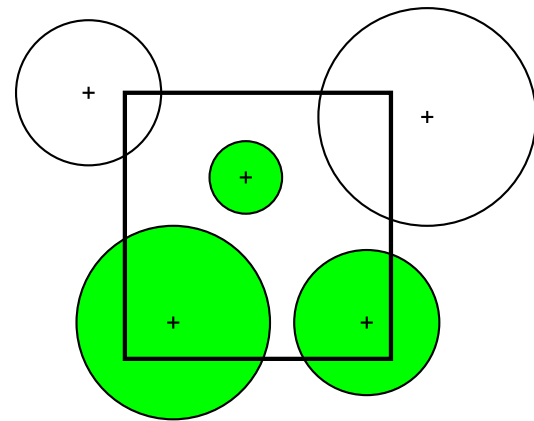
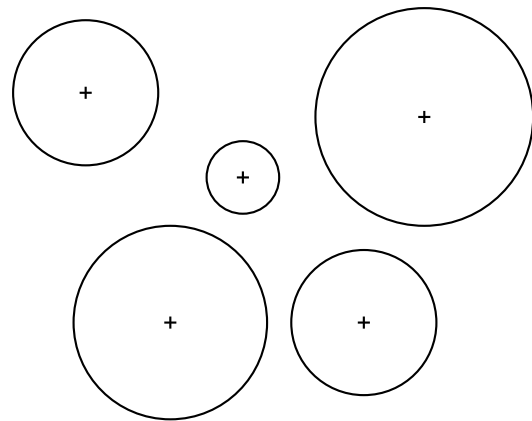
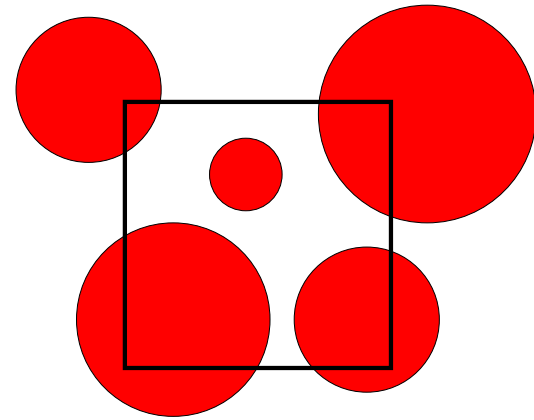
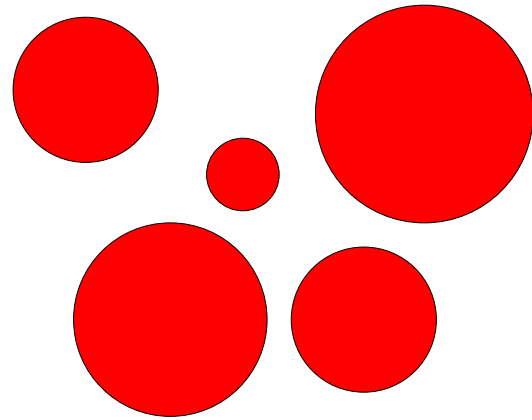


## Más sobre “número”





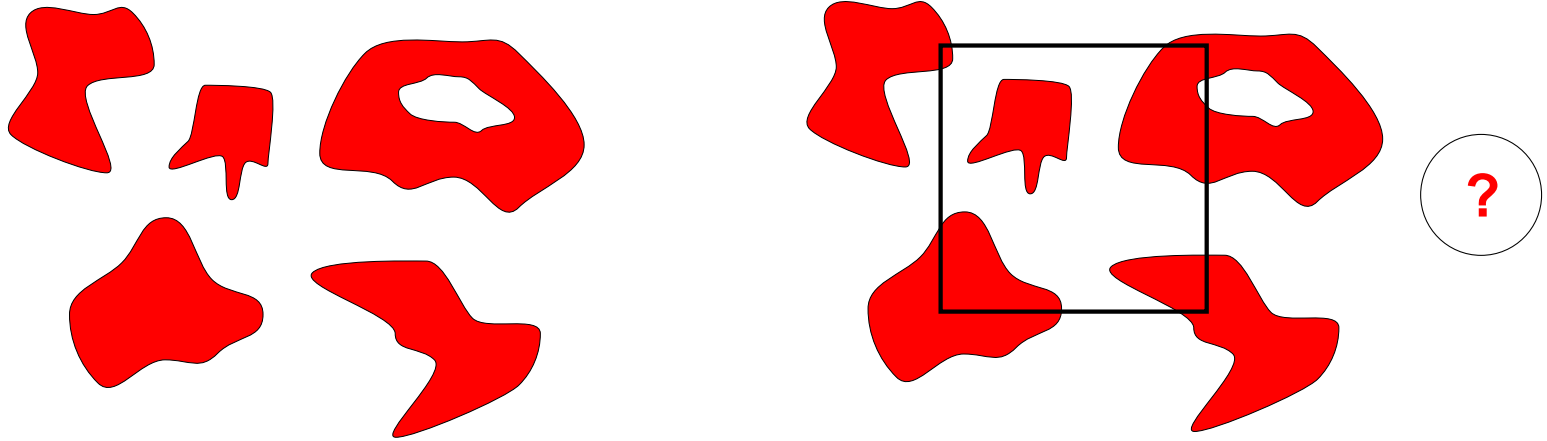


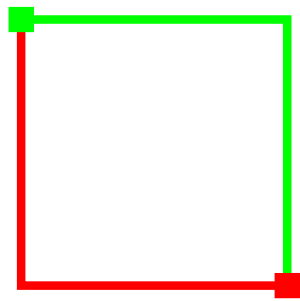
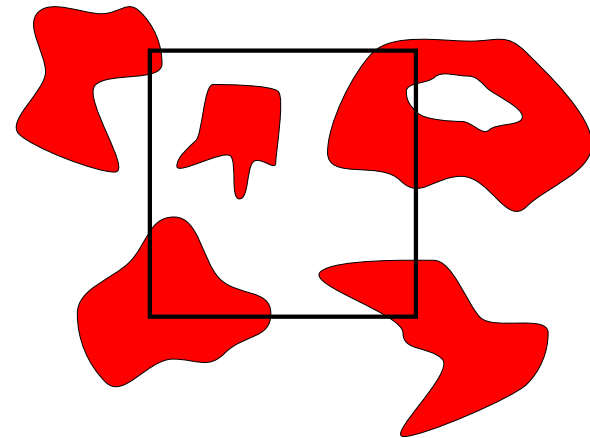
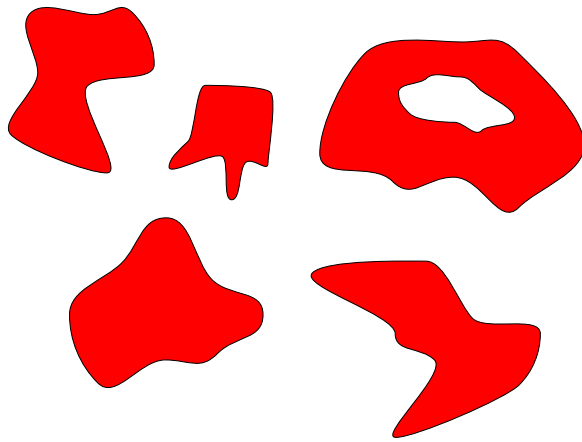


***Associated Point Rule***

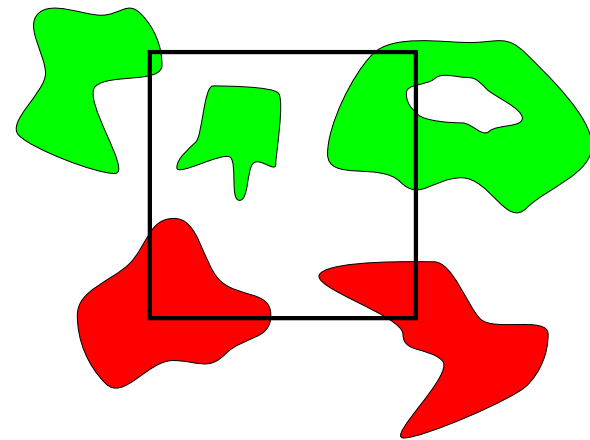
Miles R.E. (1974) In: *Stochastic Geometry*, ed. by E.F. Harding & D.G. Kendall, Wiley.

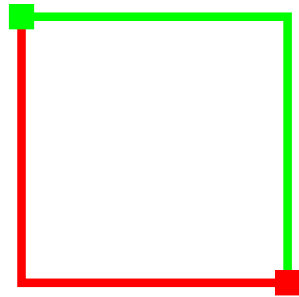
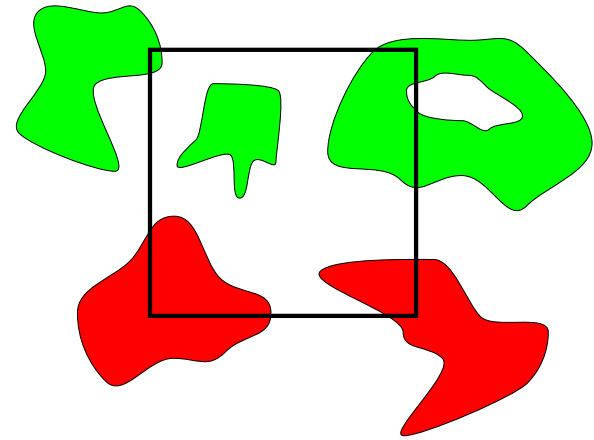
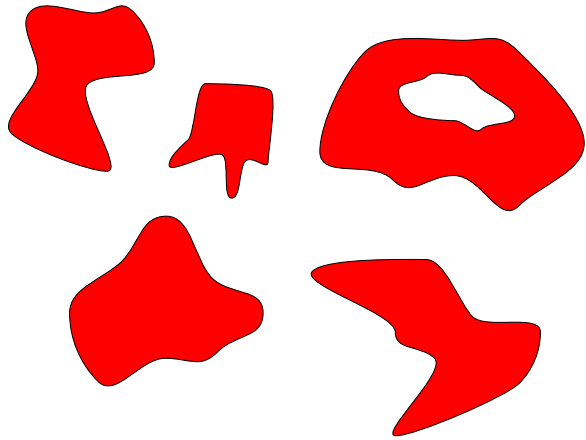
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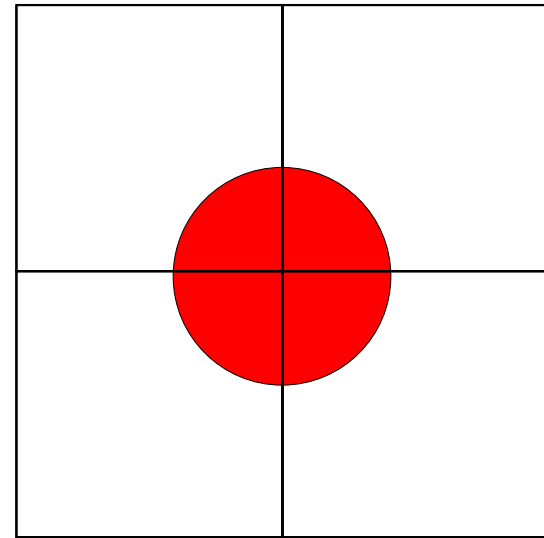


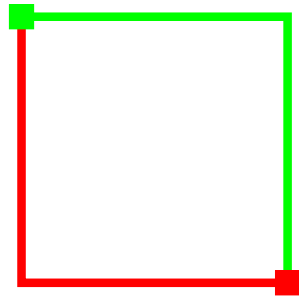
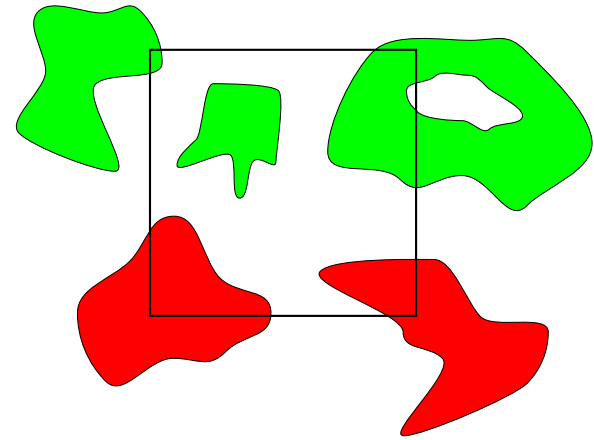
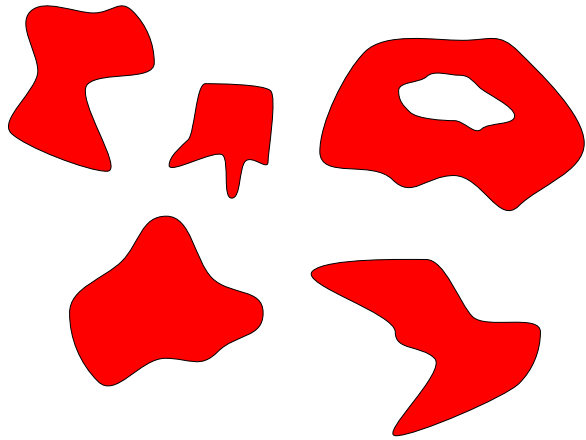
A traditional Counting Rule



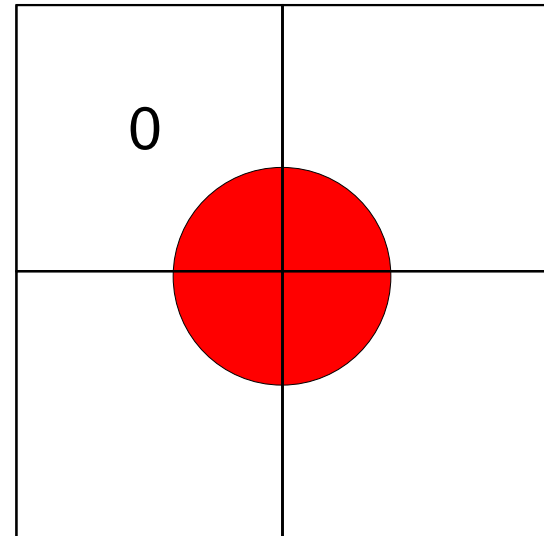


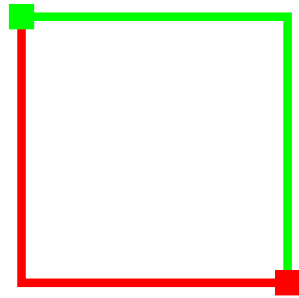
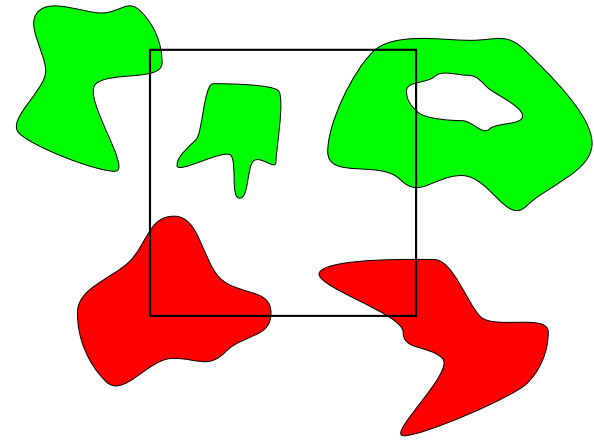
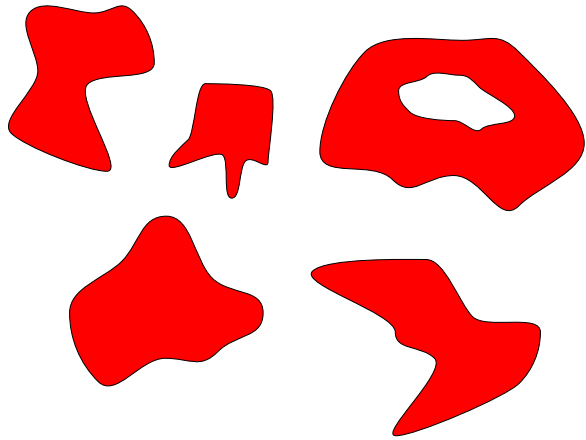
A traditional Counting Rule



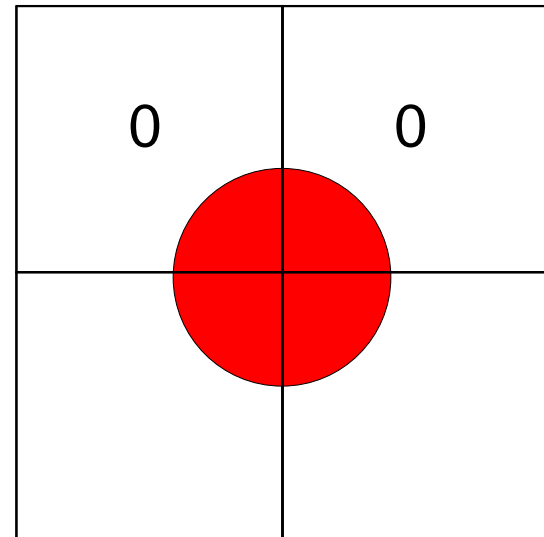


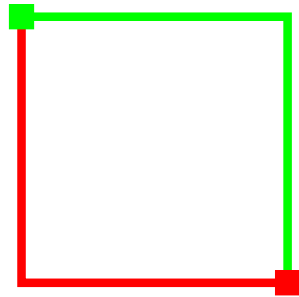
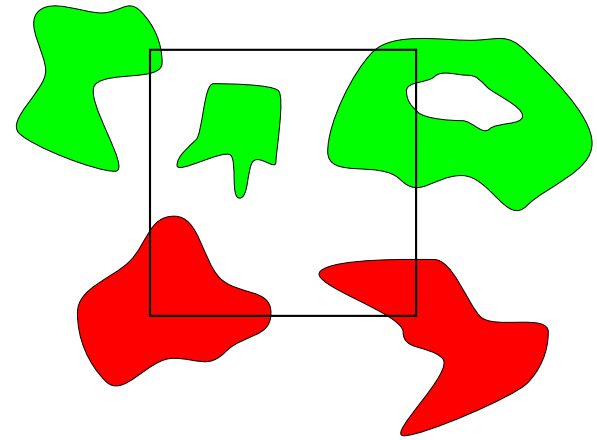
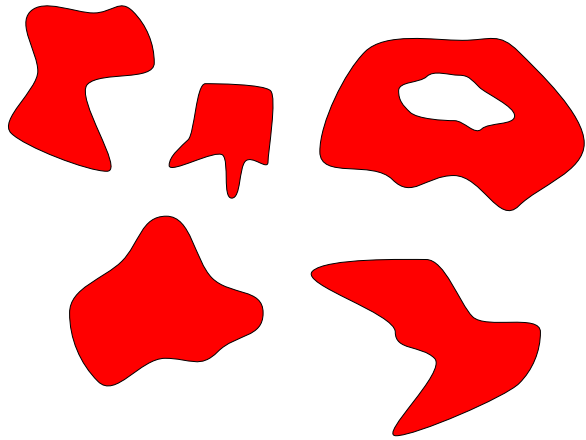
A traditional Counting Rule





A traditional Counting Rule

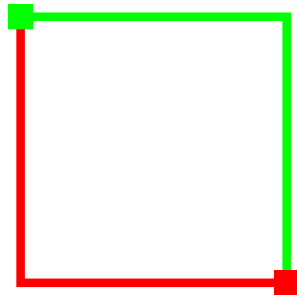
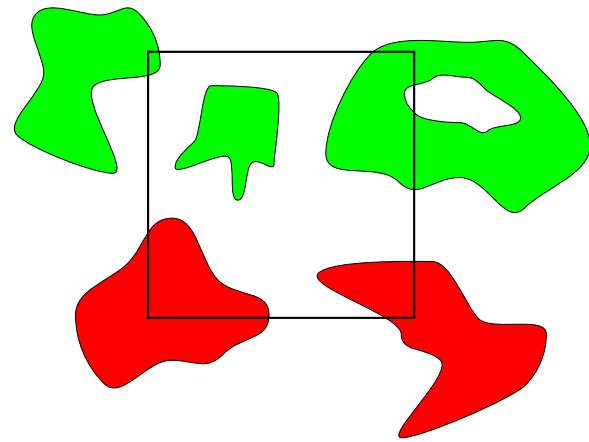
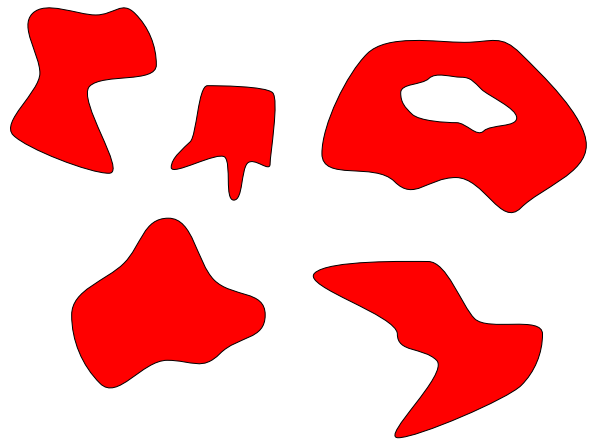




A traditional Counting Rule

0	0
1	



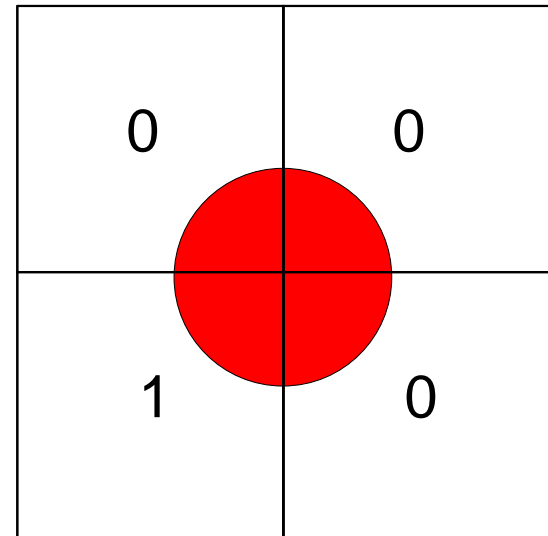
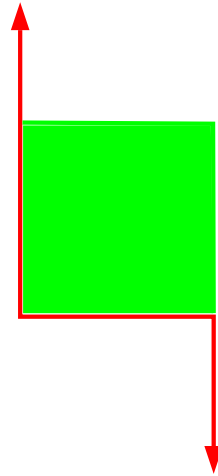
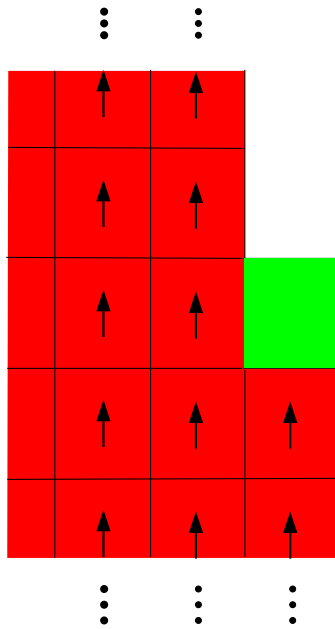


A traditional Counting Rule

0	0
1	1



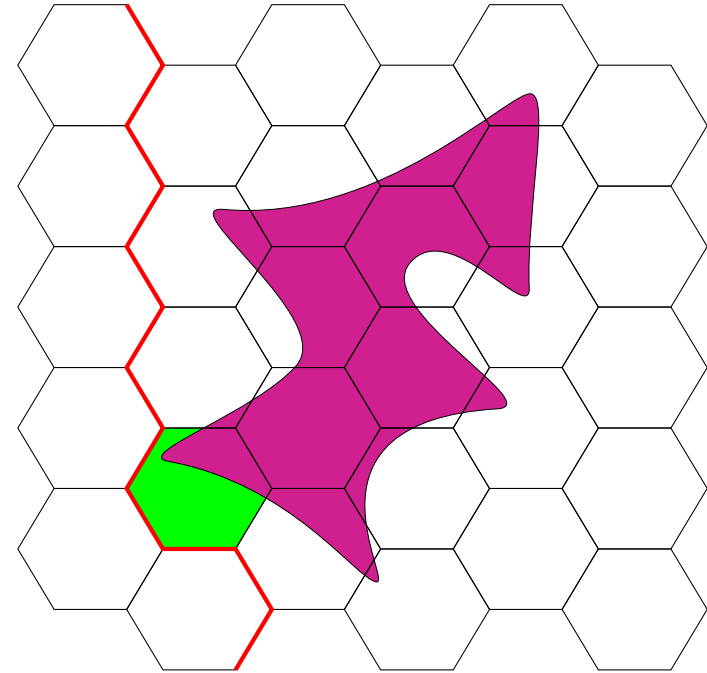
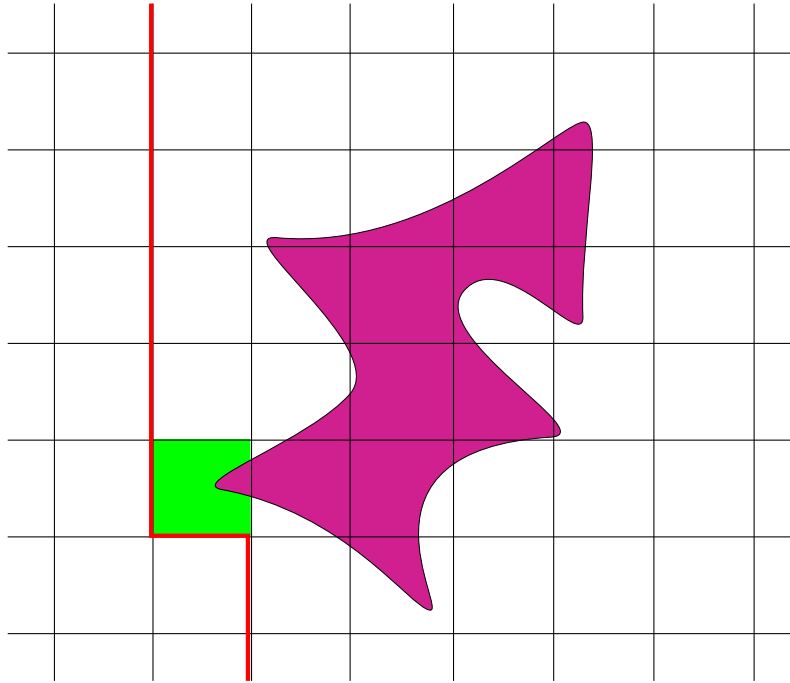
## Más sobre “número” (cont.)



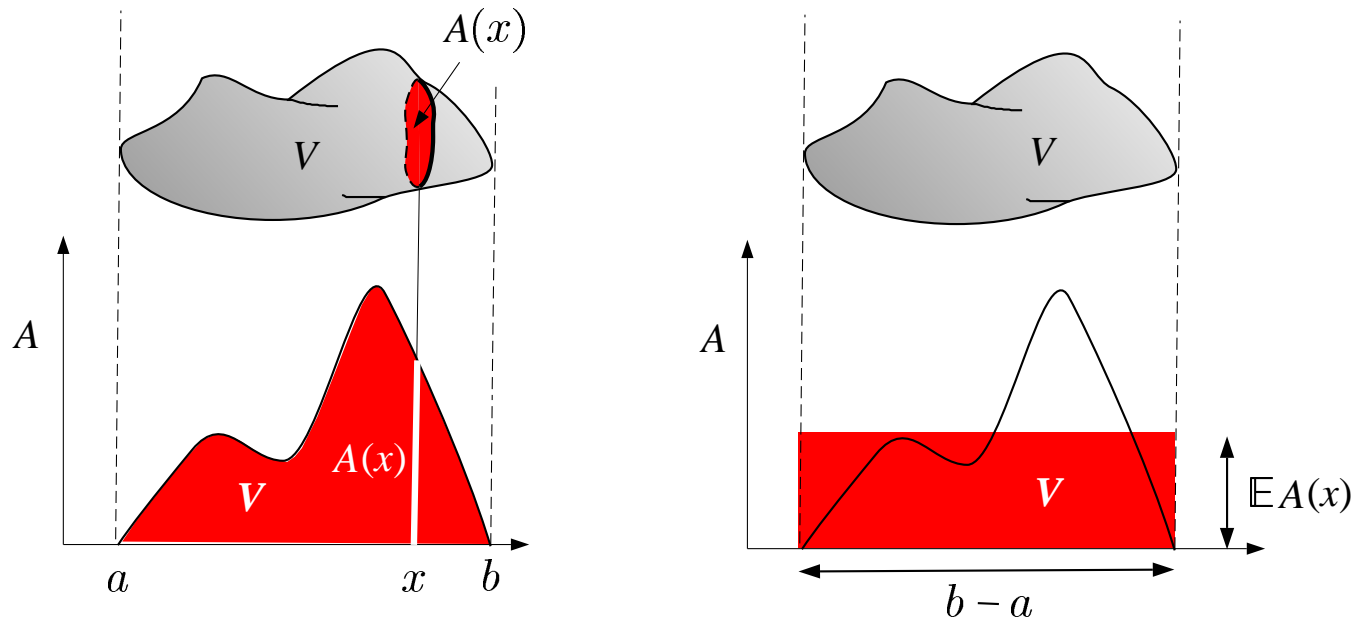
### ***Unbiased Frame*** or ***Forbidden Line Rule***

Gundersen, H.J.G. (1977) J. Microscopy 111, 219-223.

## Más sobre “número” (fin)



## Volumen: Diseño de Cavalieri

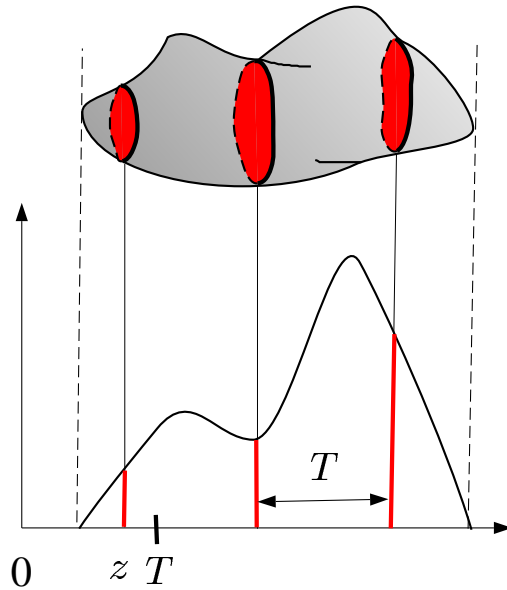


$$V = \int_a^b A(x) dx$$

$$V = (b - a) \cdot \mathbb{E} A(x)$$

$\Rightarrow$   $\hat{V} = (b - a) \cdot A(x)$  es un estimador insesgado de  $V$

## Volumen: Estimador de Cavalieri



Utilizando observaciones sistemáticas a distancia  $T$  :

$$\hat{V} = T \cdot (A_1 + A_2 + \dots + A_n)$$

es un estimador insesgado de  $V$  .

Condición: Comienzo uniforme aleatorio en  $(0, T)$ .

For an arbitrary triaxial ellipsoid,

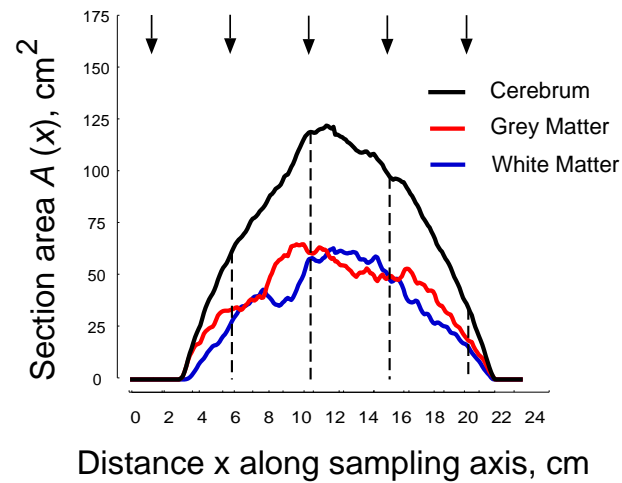
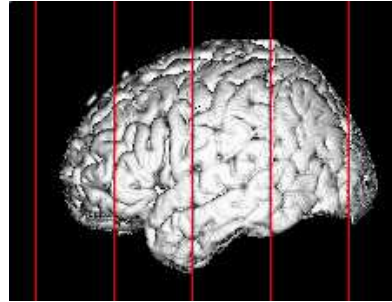
$$CE(\hat{V}) \approx \frac{1}{\sqrt{10} \cdot n^2}.$$

For  $n = 3$  sections,  $CE(\hat{V})\% \approx 3.5\%$ .

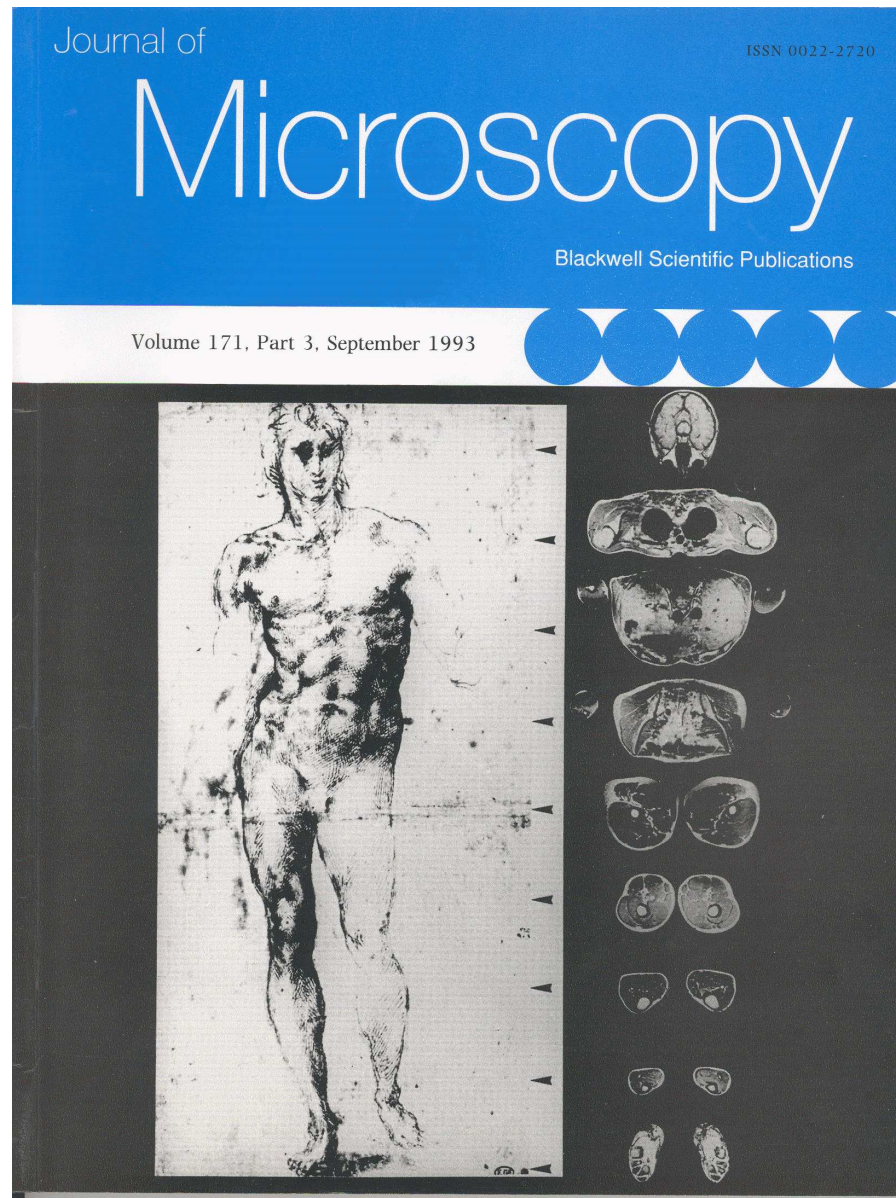
Matheron, G. (1971) The Theory of Regionalized Variables. *Cahier 5*, ENSM Paris.

# Volumen: Diseño de Cavalieri. Aplicación

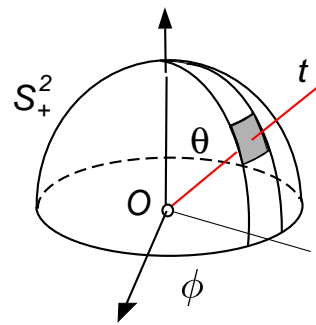
McNulty, V. *et al.* (2000) *J. Comp. Assisted Tomography* 24, 466–477



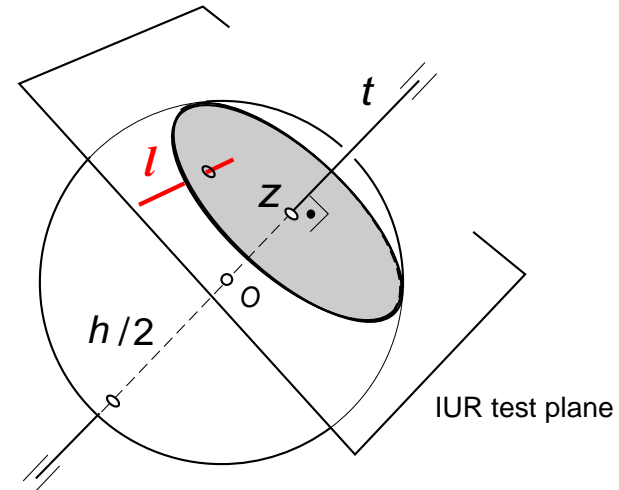
## Volumen: Diseño de Cavalieri. Aplicación



## Longitud de una curva: Diseño isótropo



Isotropic orientation



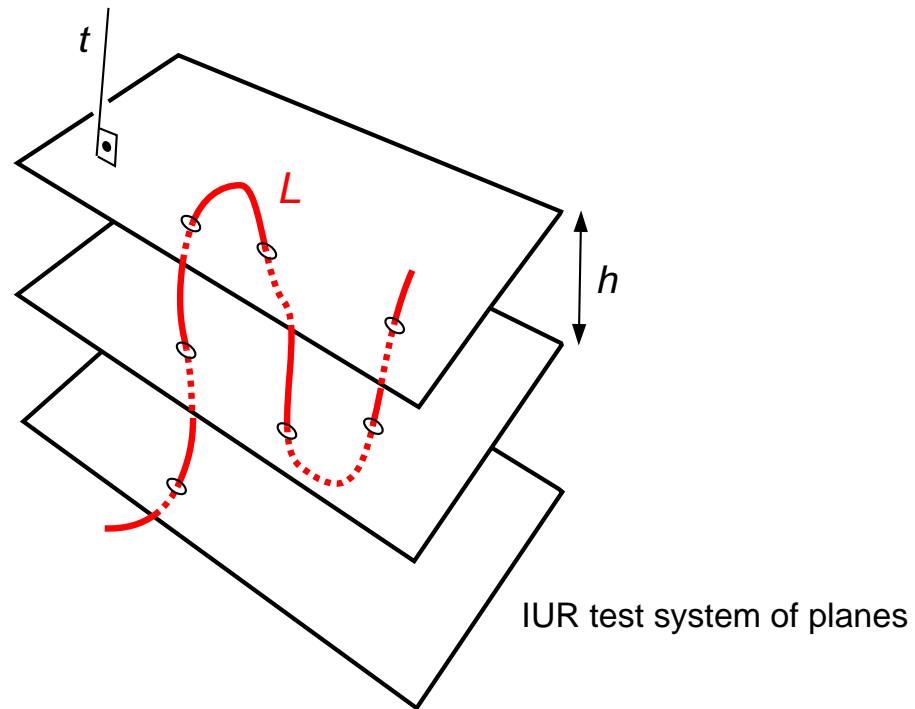
$$\mathbb{P}(dz, dt) = \frac{dz}{h} \cdot \frac{dt}{2\pi}, \quad (dt = \sin \theta d\theta d\phi),$$

$$\mathbb{P}(\text{plane hits segment}) = \mathbb{E}Q = \frac{l}{2h} \quad \left( = \frac{\pi l}{2\pi h} \right),$$

$$\Rightarrow l = 2h \cdot \mathbb{E}Q$$



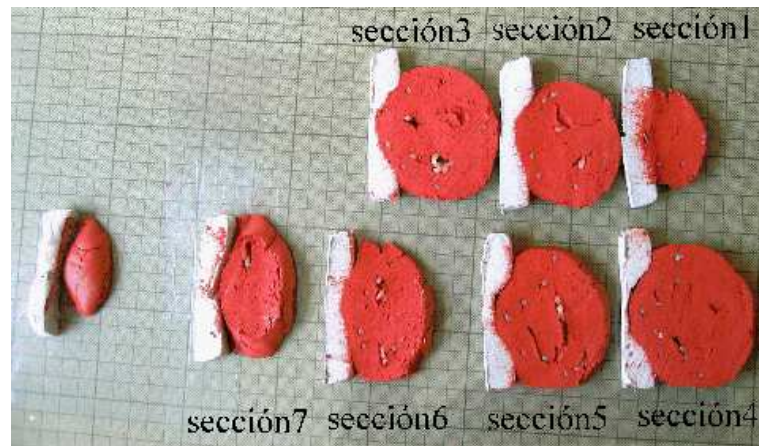
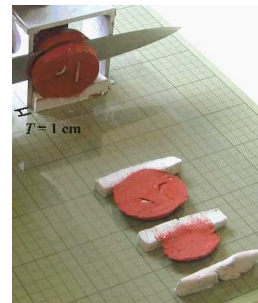
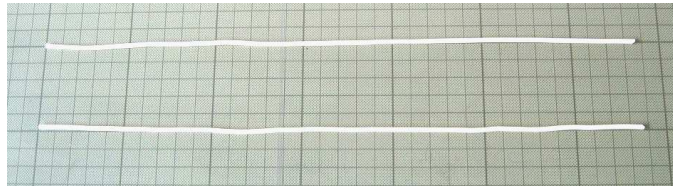
## Longitud de una curva: Diseño de Cavalieri isótopo (cont.)



$$L = 2h \cdot EQ,$$

$$\Rightarrow \hat{L} = 2hQ \quad (= 14h \text{ in the example}).$$

## Longitud de una curva: Diseño de Cavalieri isótopo. Ejemplo



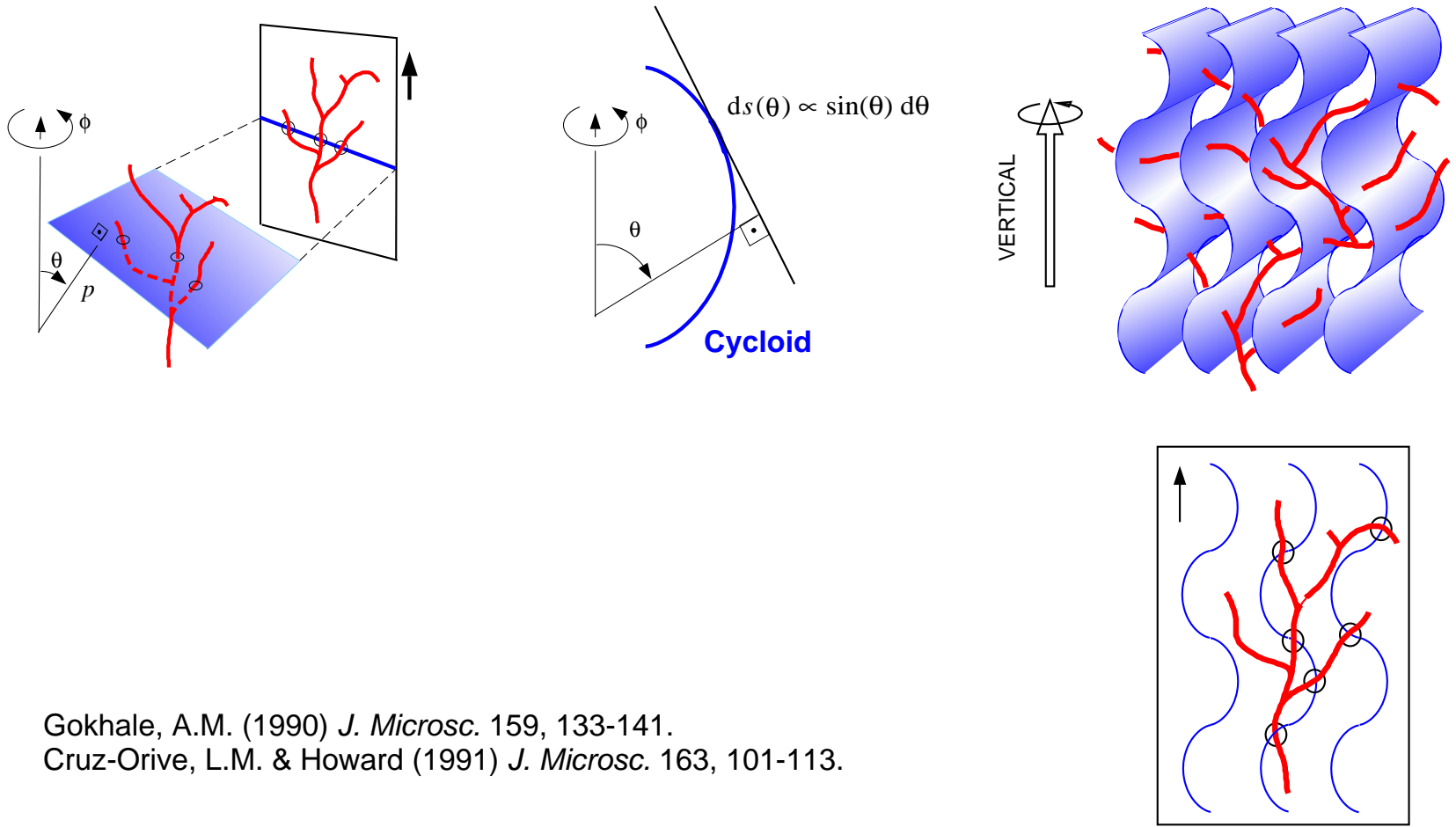
*Example.* Three boiled spaghettis of  $L = 100$  cm total length were arbitrarily bent and embedded into a 'sphere' of red plasticine. This sphere was briefly rolled on the table (to achieve isotropy), then secured on a white plasticine bed, and sliced with parallel sectioning planes  $h = 1$  cm apart, with a uniform random start in  $(0, h)$ . In the sections, a total of  $Q = 49$  transects were counted.

$$\hat{L} = 2 h Q = 2 \cdot 1 \cdot 49 = 98 \text{ cm}$$

Sagasetta, María (2005) *Estereología: Metodología Básica y Ejemplos*. Trabajo Dirigido, MATESCO, UC.

# Longitud de una curva: Diseño vertical

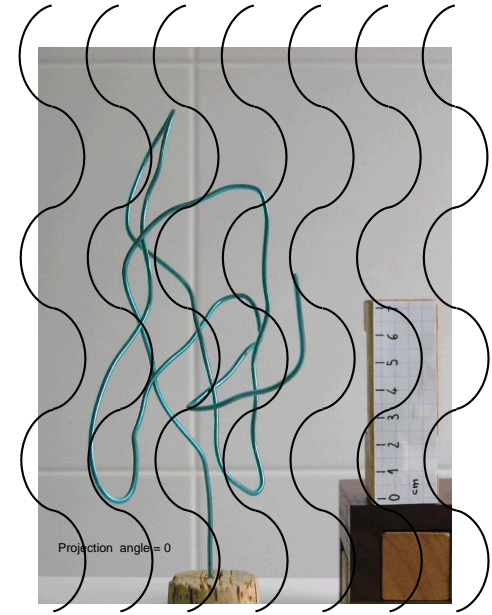
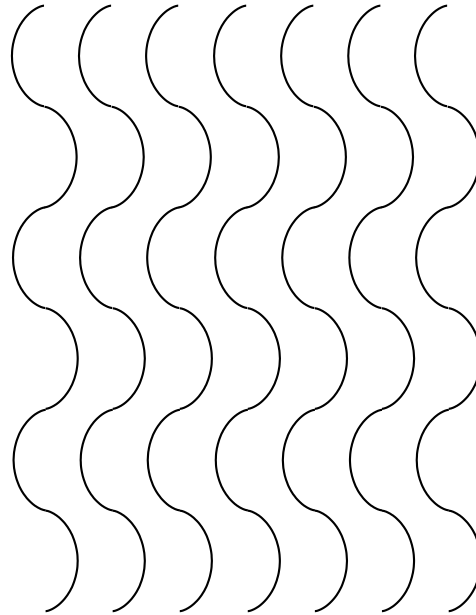
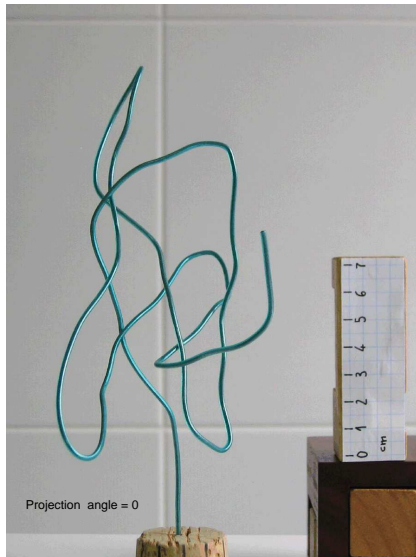
Total Vertical Projection



Gokhale, A.M. (1990) *J. Microsc.* 159, 133-141.  
 Cruz-Orive, L.M. & Howard (1991) *J. Microsc.* 163, 101-113.

$$L = 2 \cdot \frac{a}{l} \cdot \mathbb{E}Q, \quad \Rightarrow \hat{L} = 2 \cdot \frac{a}{l} \cdot Q \quad \left( = 2 \cdot \frac{a}{l} \cdot 6 \text{ in the example} \right).$$

## Longitud de una curva: Diseño vertical. Ejemplo



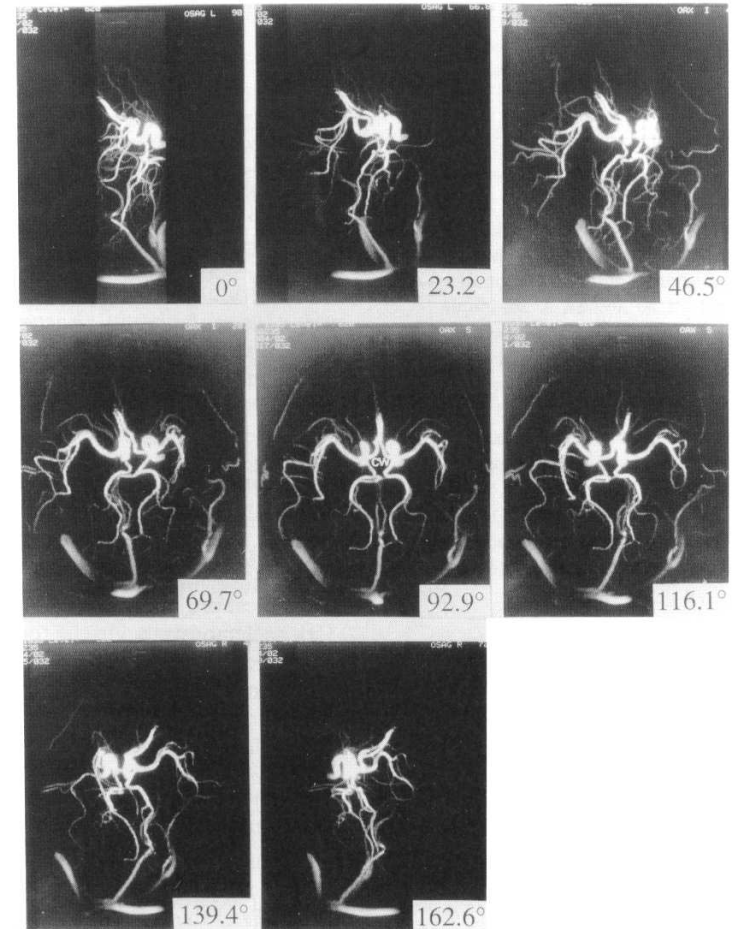
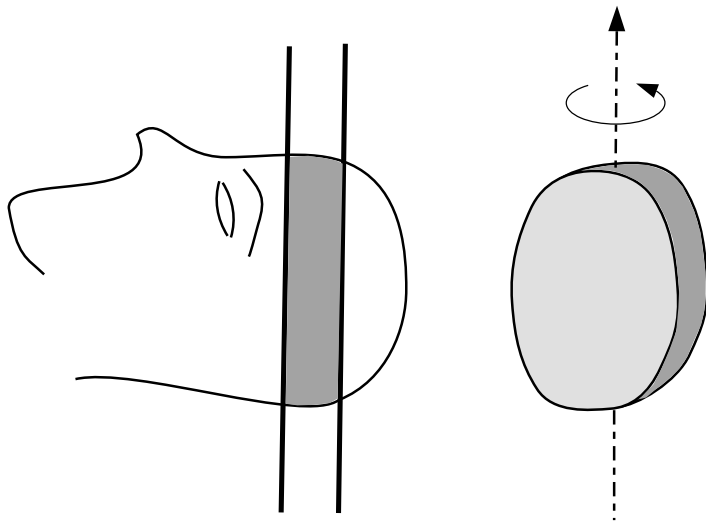
$$\hat{L} = 2 \cdot \frac{a}{l} \cdot Q$$

$$\approx 3.73 \times 23$$

$$\approx 86 \text{ cm.}$$

(True wire length = 90 cm).

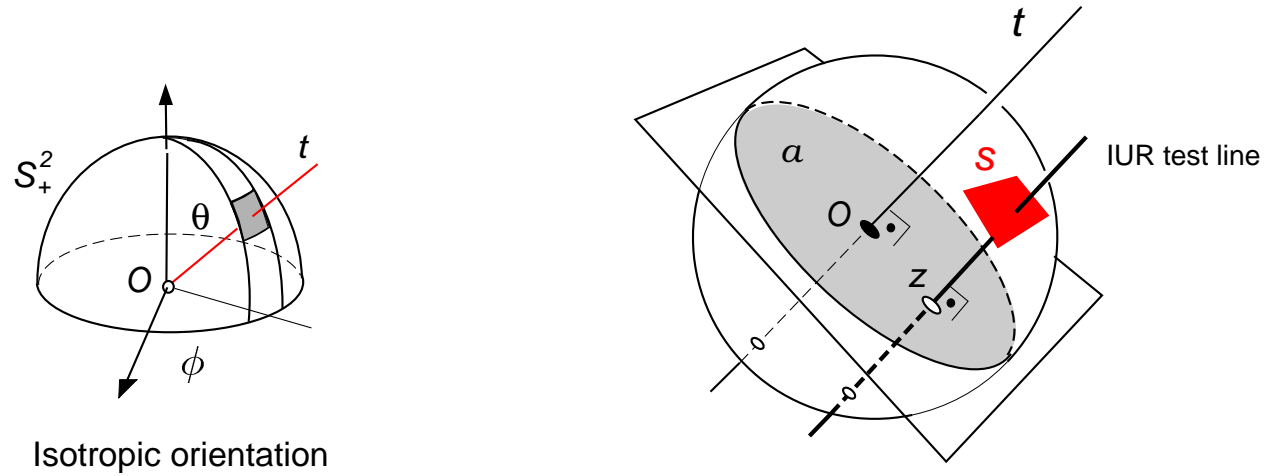
# Longitud de una curva: Diseño vertical. Aplicación a la Angiografía



Roberts, N. *et al.* (1991)  
*Magnetic Resonance Imaging* 9, 917-925.

Fig. 5. Set of MRTPVs obtained by projecting 3D phase contrast MR angiography data, of a 5-cm thick slice through the head of an adult male, at  $180^\circ/31$  intervals about a fixed vertical axis. The FOV of the images is 20 cm. Length estimates of the Circle of Willis (CW) and of the vessels are displayed in Table 4.

## Area de una superficie: Diseño isótropo



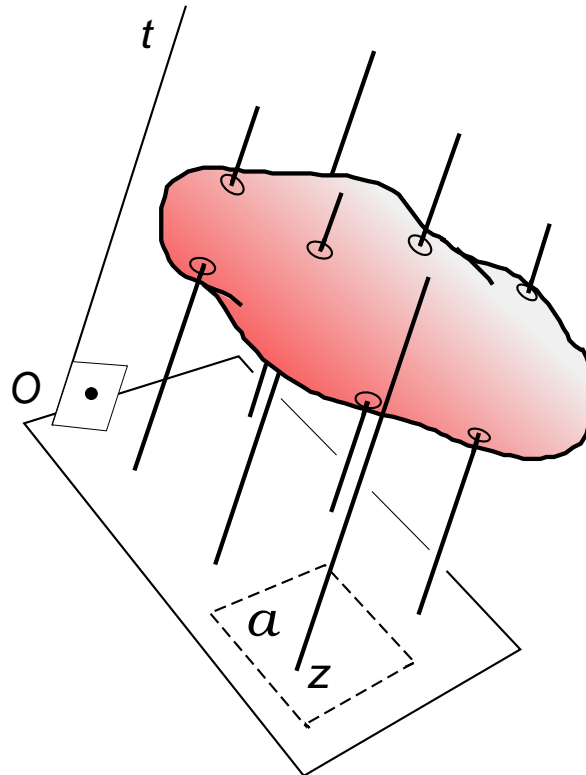
Isotropic orientation

$$\mathbb{P}(dz, dt) = \frac{dz}{a} \cdot \frac{dt}{2\pi}, \quad (dt = \sin \theta d\theta d\phi),$$

$$\mathbb{P}(\text{plane hits platelet}) = \mathbb{E}I = \frac{s}{2a} \quad \left( = \frac{\pi s}{2\pi a} \right),$$

$$\Rightarrow s = 2a \cdot \mathbb{E}I$$

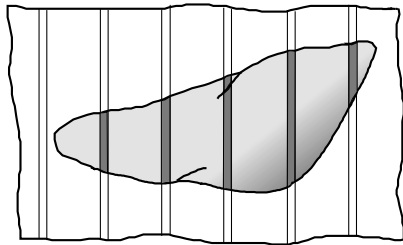
## Area de una superficie: Diseño isótropo. Sonda del Fakir



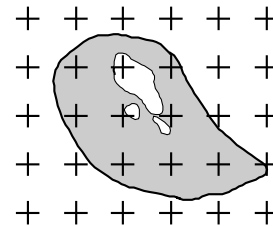
$$S = 2 \cdot a \cdot EI, \text{ in general } S = 2 \cdot \frac{v}{l} \cdot EI$$

# Area de una superficie: Diseño isótropo. Aplicación al pulmón

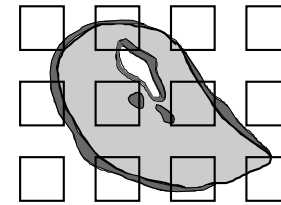
## STEREOLOGICAL MULTISTAGE DESIGN FOR LUNG



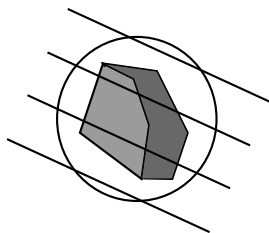
Thin and thick slices



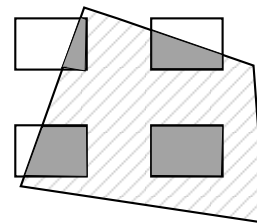
Volume by Cavalieri from thin slices



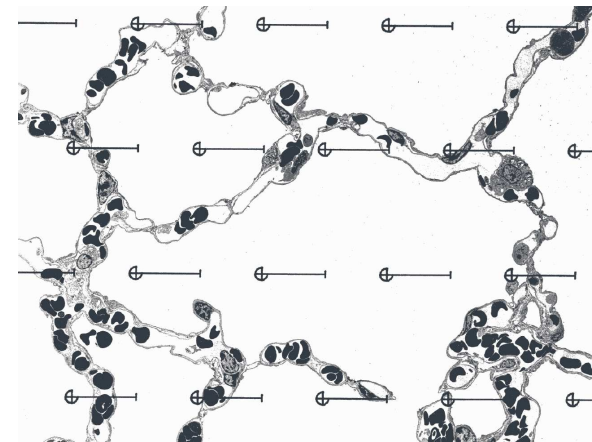
Systematic blocks from thick slices



Isotropic block sectioning for Electron Microscopy



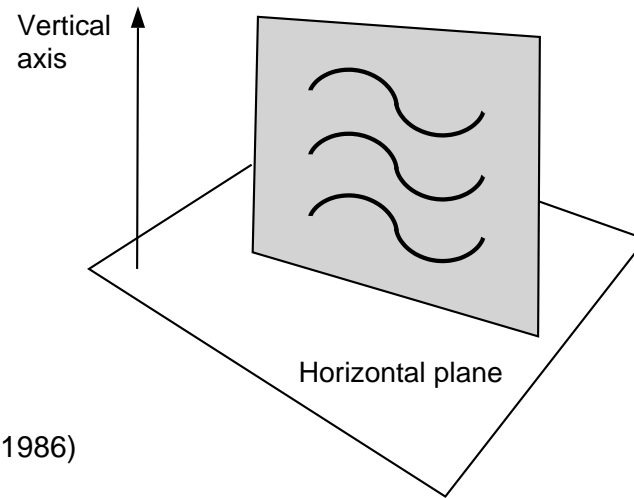
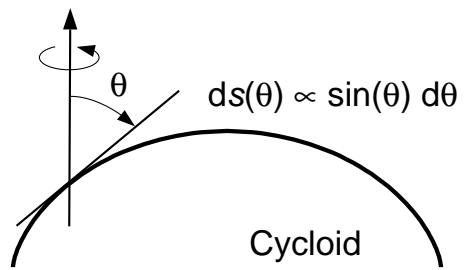
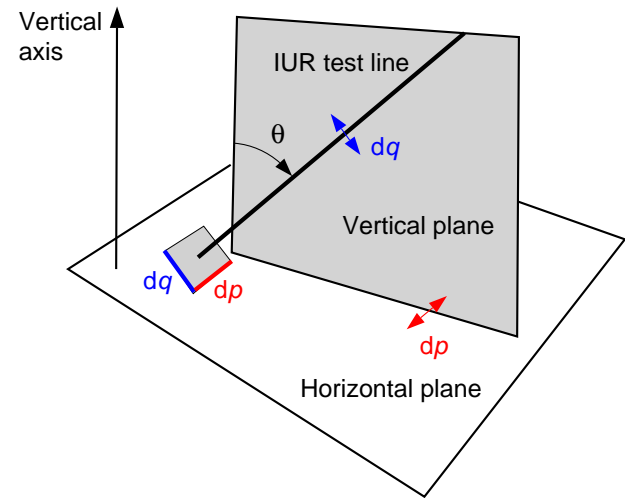
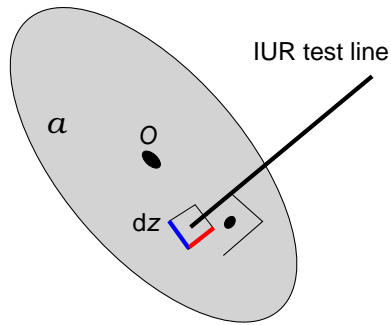
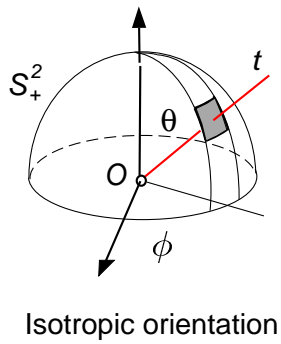
Quadrat subsampling from each EM section



EM picture with overlaid test system for point and intersection counting

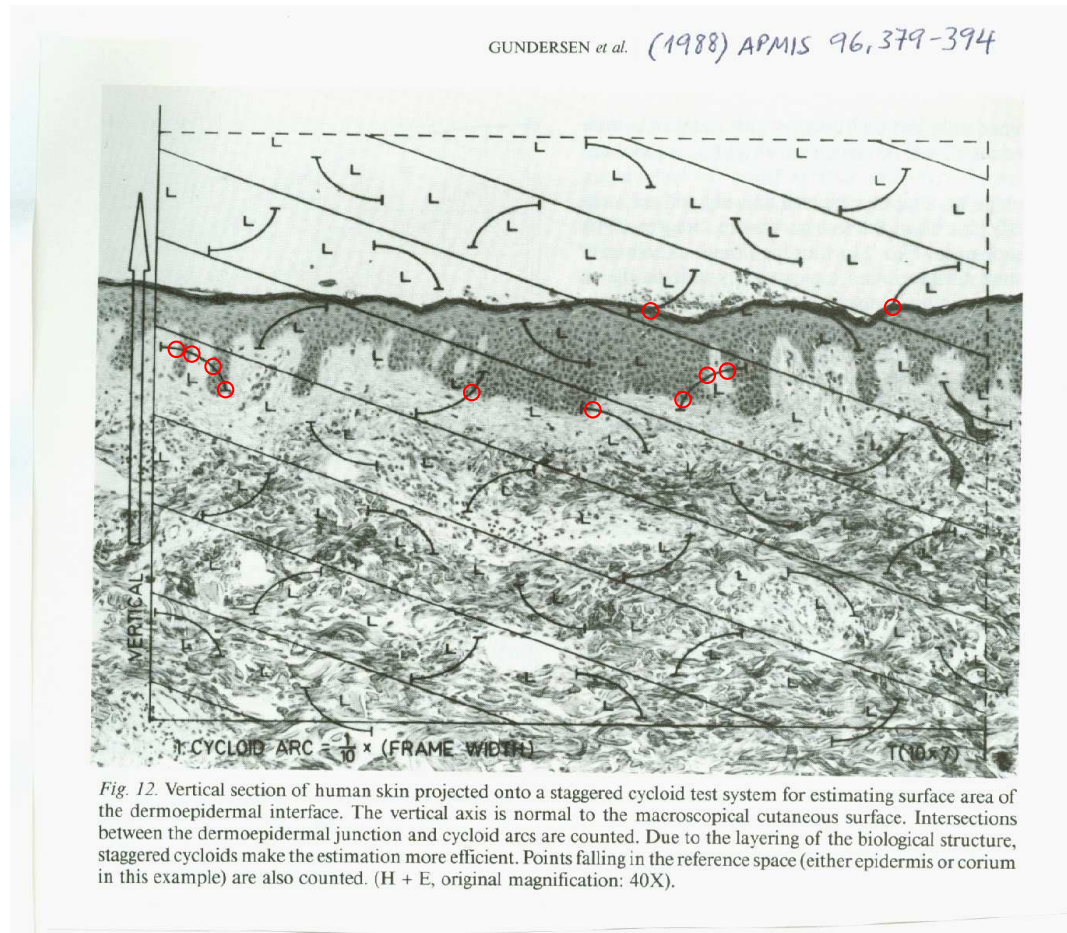


# Area de una superficie: Diseño vertical



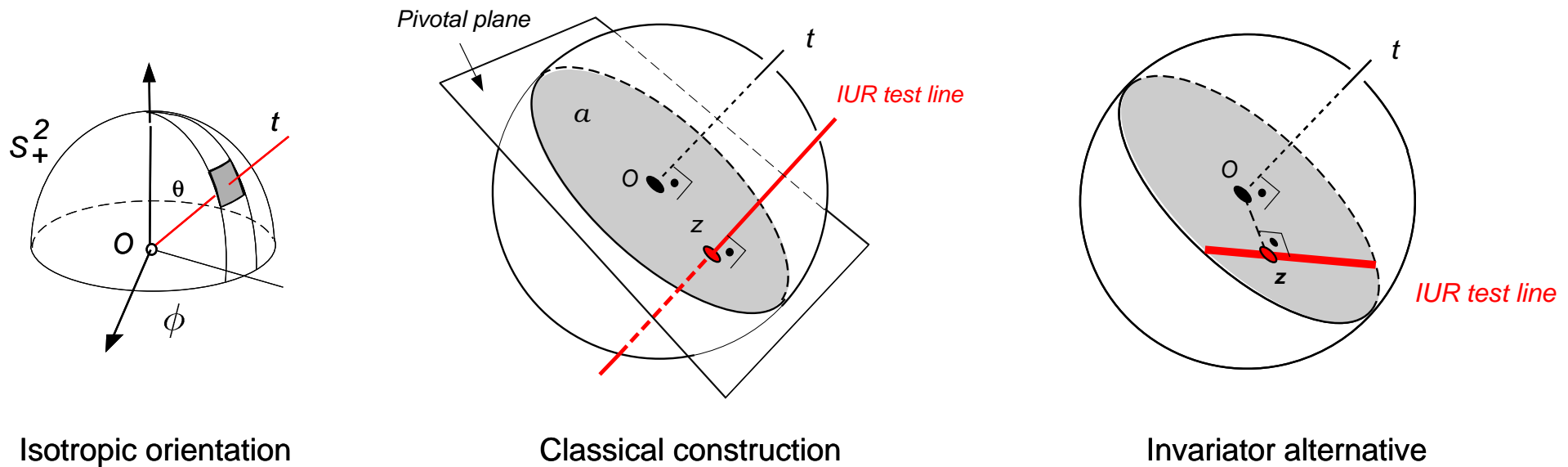
Baddeley, A., Gundersen, H.J.G. & Cruz-Orive, L.M. (1986)  
*J. Microscopy* 142, 259-276

# Area de una superficie: Diseño vertical. Aplicación en Dermatología



$$\frac{\widehat{S}(\text{dermis})}{\widehat{S}(\text{epidermis})} = \frac{c \cdot I(\text{dermis})}{c \cdot I(\text{epidermis})} = \frac{9}{2} = 4.5$$

## Invariant construction of a motion invariant test line

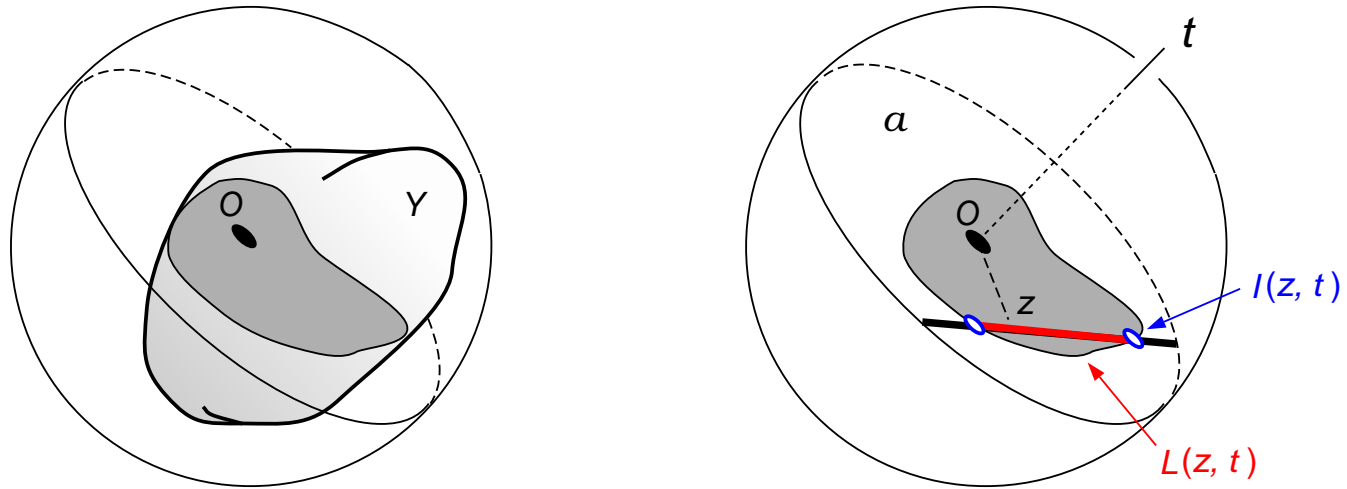


Cruz-Orive, L.M. (2005) *J. Microscopy* 219, 18–28.

Gual-Arnau X. & Cruz-Orive L.M. (2009) *Differential Geometry and its Applications* 27, 124–128.

Gual-Arnau X., Cruz-Orive L.M. & Nuño Ballesteros, J.J. (2010) *Advances in Applied Mathematics* 44, 298–308.

## Surface area and volume with the invariator

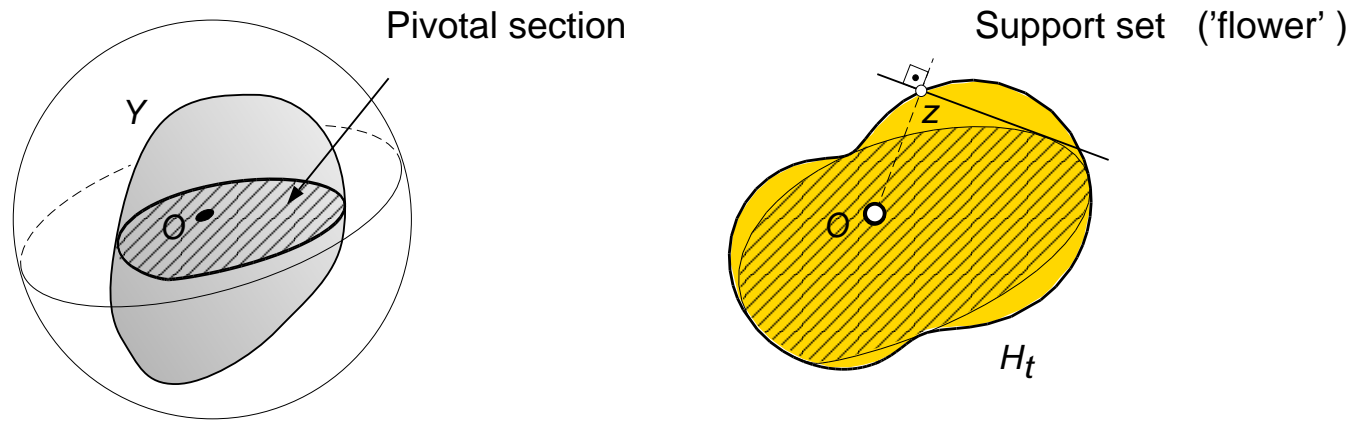


Crofton formulae:

$S(\partial Y) = 2a \mathbb{E} I(z, t)$ , where  $I(z, t) :=$  number of intersections,

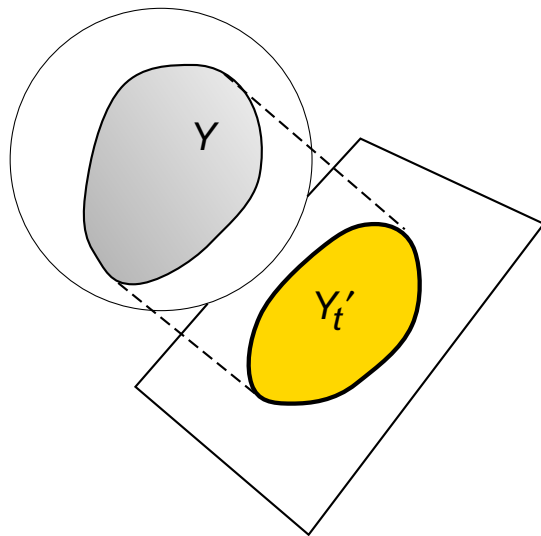
$V(Y) = a \mathbb{E} L(z, t)$ , where  $L(z, t) :=$  intercept length .

# Case of a convex particle: Surface area from the 'FLOWER' of a pivotal section



$$S(\partial Y) = 4 \mathbb{E}A(H_t).$$

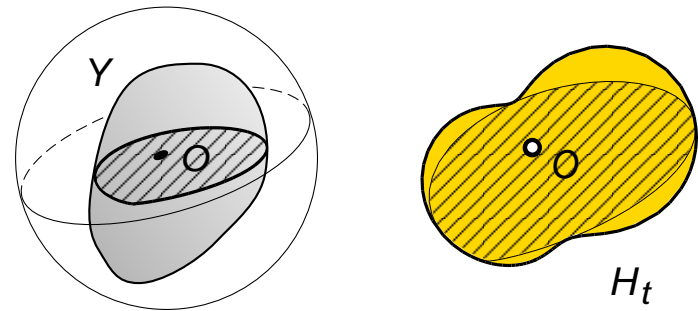
## A duality



$$S(\partial Y) = 4 \mathbb{E} A(Y'_t)$$

Cauchy projection formula

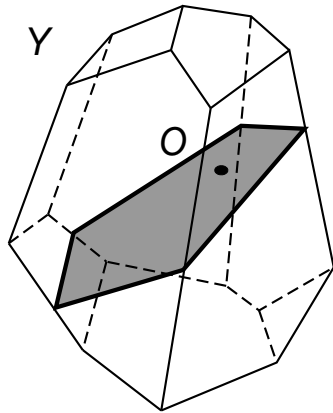
Y convex



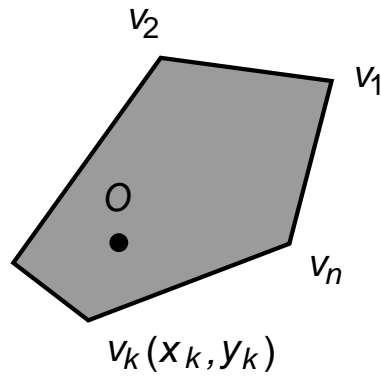
$$S(\partial Y) = 4 \mathbb{E} A(H_t)$$

Flower formula

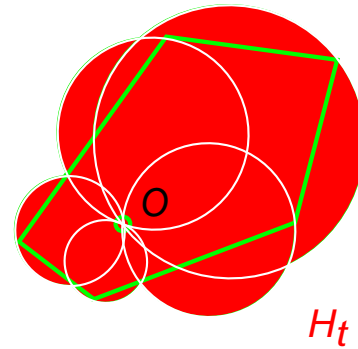
## Flower area of a convex polygon



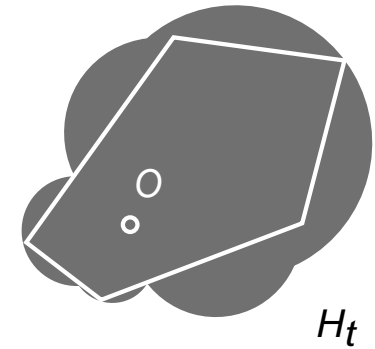
Convex polyhedron



Pivotal section



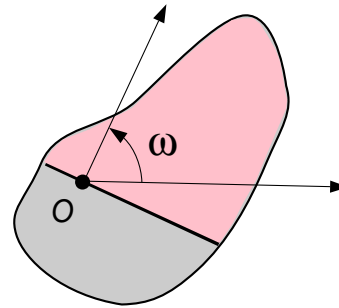
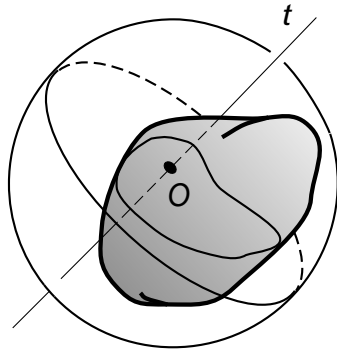
Flower



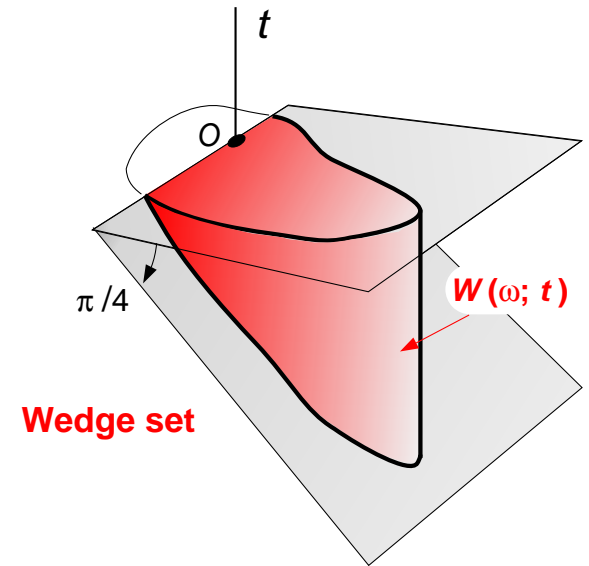
$H_t$

Cruz-Orive L.M. (2011) *J. Microsc.* 243, 86–102.

# Volume of an arbitrary particle via the mean WEDGE volume



Pivotal section

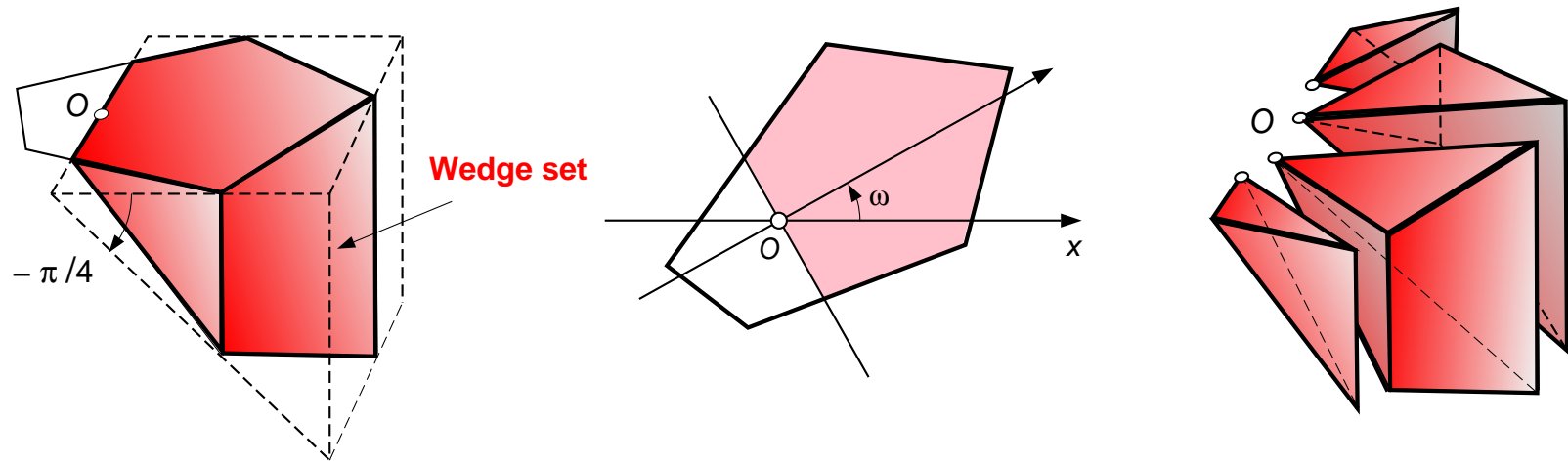


Wedge set

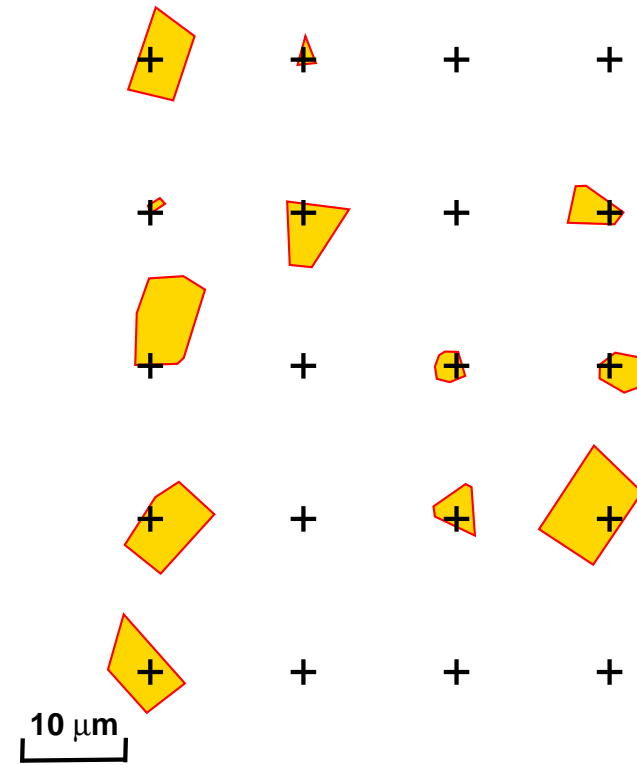
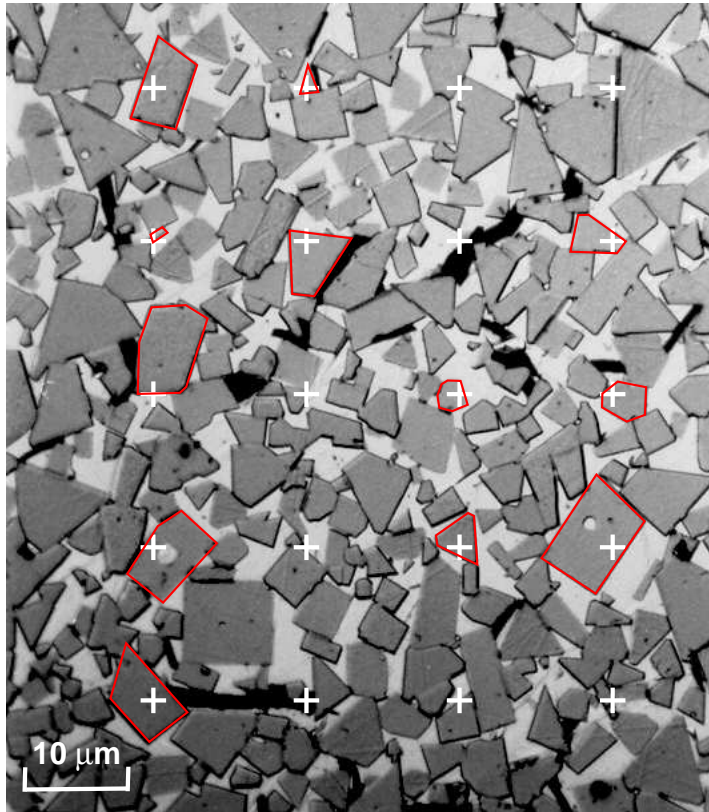
$$V(Y) = 2\pi \mathbb{E}_t V(W_t), \quad V(W_t) := \mathbb{E}_\omega V\{W(\omega; t)\}$$



# Mean wedge volume of a convex polygon



# Mean volume weighted surface area and volume of cemented carbide grains



$$\mathbb{E}_V(X) = \frac{\mathbb{E}_N(VX)}{\mathbb{E}_N(V)}, \text{ here we want } \mathbb{E}_V(S), \mathbb{E}_V(V).$$

$$\bar{s}_V = 4\bar{F}, \quad \bar{F} := \text{Mean flower area,}$$

$$\bar{v}_V = 2\pi\bar{W}, \quad \bar{W} := \text{Mean wedge volume.}$$

## Mean volume weighted surface area and volume of cemented carbide grains: Results

Quadrat	# polygons	$\bar{A}, \mu\text{m}^2$	$\bar{s}_V = 4\bar{F}, \mu\text{m}^2$	$\bar{v}_V = 2\pi\bar{W}, \mu\text{m}^3$
1	20	21.24	144.9	135.5
2	17	32.36	209.3	233.6
3	16	24.76	160.1	160.3
4	17	21.44	142.5	124.9
Pool	70	24.80 (10.4%)	163.4 (9.5%)	162.4 (15.0%)
Results of 1997			$\bar{s}_V \mu\text{m}^2, \text{ surfactor}$	$\bar{v}_V \mu\text{m}^3, \text{ PSI}$
			154.5 (4.4%)	161.3(23.7%)

The 1997 results were reported in Karlsson & C–O (1997). For the “surfactor” and “PSI” (point sampled intercepts) methods, see the mentioned study.

## Algunas conclusiones

- La Estereología es a menudo la única disciplina disponible para estimar cantidades geométricas.
- No es necesario modelar la forma de los objetos.
- Muestrear es más importante que medir.
- Los métodos están casi siempre motivados por las ciencias naturales.